Abstract

On Partitions of F_q^n into Perfect Codes

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The problem of the enumeration and the classification of all partitions of the set F_q^n of all q-ary ($q \geq 2$) vectors of length n into perfect codes is closely linked to the classical problem of classifying all perfect codes. Partitions of $\mathbb{F}_2^n$ are closely related to the important vertex-coloring problem of $\mathbb{F}_2^n$ into codes with prescribed distance. Each partition can generate a coloring, concerning the study of scalability of optical networks, or a perfect coloring, called also a partition design or equitable partition.

A code C is a *perfect binary code correcting single errors* (briefly a perfect code) if for any vector $x \in F_q^n$ there exists exactly one vector $y \in C$ such that $d(x, y) \leq 1$. Two partitions of F_q^n into codes are called *different* if they differ in at least one code. Two partitions we call *equivalent* if there exists an isometry of the space F_q^n that transforms one partition into another one.

In [1] the amount of different partitions of F_2^n into perfect codes of length n was proven to be at least

$$2^{2^{\frac{(n-1)}{2}}} \tag{1}$$

for every admissible $n \geq 31$. We validate the estimation (1) for every admissible $n \geq 7$. In [2] two constructions of partitions of the space F_q^n into perfect q-ary codes of length n are presented and the lower bound on the number of such different partitions is given. Several constructions of transitive partitions of F_2^n into codes were done in [3]. The approach is developed in [4] to construct 2-transitive and vertex-transitive partitions of F_2^n into perfect binary codes. The lower bounds on the number of nonequivalent such partitions are done.

The following result is proven for the number of different partitions $\mathcal{R}_N$ of F_2^N, $N = 2^m, m \geq 4$ into extended perfect binary codes:

$$\mathcal{R}_N \geq 2^{2^{\frac{N}{2}}} \cdot 2^{2^{\frac{N-4}{4}}}.$$

References

1. *F. I. Solov'eva* "On perfect codes and related topics," Com^{2}Mac Lecture Note. Series 13. Pohang, 2004. 80 p.

2. *F. I. Solov'eva, A. V. Los'* "Constructions of partitions of F_q^n into perfect q-ary codes," Discrete Analysis and Oper. Research, **16** (3) (2009), 63–73 (in Russian).

3. *F. I. Solov'eva* "On transitive partitions of n-cube into codes," *Problems of Inform. Transm.*, **45** (1) (2009), 27-35.

4. *F. I. Solov'eva, G. K. Guskov* "On the constructions of the vertex-transitive partitions of F^n into perfect codes," *Siberian Mathematical Journal*, accepted, 2010.