Abstract

Completely regular codes in Johnson graph $J(2w+1, w)$ with covering radius 1

Sergey V. Avgustinovich, Ivan Yu. Mogilnykh
Sobolev Institute of Mathematics, Novosibirsk
Novosibirsk State University, Russia
ivmog84@gmail.com, avgust@math.nsc.ru

An arbitrary collection of vertices of a graph G is called *a code* in graph G. A code C in graph G is *completely regular* (by Delsarte [2]) if the number of elements of the code C that are at distance i from arbitrary vertex v of G is uniquely determined by the distance from v to C. *Covering radius* ρ of the code C is the maximum possible distance between a vertex of G and C.

In this talk we consider completely regular codes with covering radius $\rho = 1$ in Johnson graph $J(n, w)$. This class of codes includes such objects as a 1-perfect constant weight codes and designs with the largest nontrivial strength (namely $(w-1)-(n, w, \lambda)$-designs). The strength of any perfect code in Johnson graphs (as a t-design) can be calculated using the result by Etzion and Schwarz [3]. In [4] the generalization of this result for all completely regular codes in Johnson graph with covering radius 1 was proved.

Further study of completely regular codes in Johnson graphs reveals more connections with t-designs. In this talk we show that constructions of Alltop [1] being applied to completely regular codes with $\rho = 1$ in $J(2w+1, w)$ give completely regular codes in $J(2w+2, w+1)$ with $\rho = 1$. Applying another construction to $(w-1)-(n, w, \lambda)$-designs we get two completely regular codes with $\rho = 1$ in $J(n, w-1), J(n, w+1)$. Also we show the connection between completely regular codes in Kneser (or Odd) graph and those in Johnson graphs.

References

1. *Alltop W.O.* Extending t-designs // Journal of Combinatorial Theory Ser. A. 1975. V. 18. I. 2. P. 177-186.

2. *Delsarte P.* An Algebraic Approach to the Association Schemes of Coding Theory // Philips Res. Rep. Suppl. 1973. V. 10. P. 1-97.

3. *Etzion T., Schwarz M.* Perfect Constant-Weight Codes // IEEE Trans. Inform. Theory. 2004. V. 50. N 9. P. 2156-2165.

4. *Mogilnykh I.Yu.* On the regularity of perfect 2-colorings of the Johnson graph. // Probl. Inform. Transm. 43(4), 271-277 (2007)