Abstract

Describing Polynomials as Equivalent to Explicit Solutions

Uwe Schauz

King Fahd University of Petroleum and Minerals
P.O. Box 201, Dhahran 31261, Saudi Arabia
schauz@kfupm.edu.sa

We present a coefficient formula which provides some information about the polynomial map $P|_{I_1\times\ldots\times I_n}$ when only incomplete information about a polynomial $P(X_1,...,X_n)$ is given. It is an integrative generalization and sharpening of several known results and has many applications, among these are:

1. The fact that polynomials $P(X_1) \neq 0$ in just one variable have at most $\deg(P)$ roots.
2. Alon and Tarsi's Combinatorial Nullstellensatz.
3. Chevalley and Warning's Theorem about the number of simultaneous zeros of systems of polynomials over finite fields.
4. Ryser's Permanent Formula.
5. Alon's Permanent Lemma.
6. Alon and Tarsi's Theorem about orientations and colorings of graphs.
7. Scheim's formula for the number of edge n-colorings of planar n-regular graphs.
8. Alon, Friedland and Kalai's Theorem about regular subgraphs.
9. Alon and Füredi's Theorem about cube covers.
10. Cauchy and Davenport's Theorem from additive number theory.
11. Erdős, Ginzburg and Ziv's Theorem from additive number theory.