# On the Schubert cells and Coding Theory

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Let $V = {F_q}^n$ be the vector space over the finite field $F_q$ with the standard base $e_1, e_2, \ldots, e_n$, $d(\mathrm{x}, \mathrm{y})$ is the Hamming metrics on $V$. So $d(\mathrm{x}, \mathrm{y}) = \{i | x_i \neq y_i\}$ for $\mathrm{x} = (x_1, x_2, \ldots, x_n), \mathrm{y} = (y_1, y_2, \ldots, y_n)$. The totality of linear codes is the projective geometry $P(V)$ i.e. the totality of all subspaces $W < V$, $0 < \dim(W) < n$.

The geometry $\Gamma$ of Weyl group $A_{n-1}$ is the totality of subsets of $\{1, 2, \ldots, n\}$. with the incidence relation $I$: $J_1 I J_2$ if and only if $J_1 \neq J_2$ and $J_1 \cap J_2$ coincides with $J_1$ or $J_2$. We will identify further the subset $J$ of the Weyl geometry $A_{n-1}$ subspace spanned by $e_j, j \in J$.

We define Schubert equivalence $<=>$ on the set $P(V)$ by the following way: For each $W \in P(V)$ we consider graph $\Gamma(W)$ of incidence relation $I$ with the weight function $f$ on the vertex set defined by equality $f(J) = \dim(W \cap J)$. Let us assume that $W_1 <=> W_2$ if and only if graphs with weights $\Gamma(W_1)$ and $\Gamma(W_2)$ are isomorphic.

As it follows from definition each class of equivalence is the union of small Schubert cells. The number of classes does not depend on $q$ for "sufficiently large" number $\mathrm{char}(F_q)$.

We consider the algorithm of generation of $P(V)$ in the memory of computer which costs $O(P(V))$. It based on the idea of embedding of $P(V)$ into the Borel subalgebra of simple Lie algebra $A_{n-1}$ over $F_q$ and allows to check whether or not two m-dimensional subspaces $W_1$ and $W_2$, where $m$ is constant, are Schubert equivalent for polynomial time on $n$.

Let us consider the Grassman metrics $D(x, y)$ on the totality of $m$-dimensional subspaces $G(n, m, q)$: $D(W_1, W_2) = m - \dim(W_1 \cap W_2)$. The mentioned above interpretation of the geometry $P(V)$ allows to measure the distanse between subspaces $W_1$ and $W_2$ for polynomial time in $n$.

We will use the interpretations of Lie geometries from [2] to create fast algorithms to measure the distance between vertices of distance regular graphs of Lie type and related to them Hemmeter and Ustimenko graphs, double Grassman graphs (see [1]). Distance regular graphs of Coxeter and Lie type are important examples of graphs for which there is the algorithms to measure the distance between vertices which is much faster in comparison with well known Dijkstra method.

REFERENCES:

[1] A. Brower, A. Cohen, A. Nuemaier, *Distance regular graphs*, Springer, Berlin, 1989.

[2]V. Ustimenko. On the varieties of parabolic subgroups, their generalisations and combinatorial applications, Acta Applicandae Mathematicae, 52 , 1998, 223-238.