Abstract

Local distributions of q-ary perfect codes and q-ary centered functions

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We study perfect codes with distance 3 in the space F_q^n with Hamming metric. Centered functions generalizes perfect codes: the sum of function values over an arbitrary radius 1 ball is constant (say, equals zero). The nontrivial (i.e. nonconstant) centered functions exist for $n \equiv 1 \mod q$.

Let γ be a k-dimensional subcube of F_q^n and $\mathbf{a} \in \gamma$. For a function $f : F_q^n \longrightarrow \mathbf{R}$ denote $v_i^{\mathbf{a}}(\gamma) = \sum f(\mathbf{x})$, where the summation is taken over all $\mathbf{x} \in \gamma$ at distance i from $\mathbf{a}$, $i = 0, 1, \ldots, k$. The vector $v = (v_0^{\mathbf{a}}(\gamma), v_1^{\mathbf{a}}(\gamma), \ldots, v_0^{\mathbf{a}}(\gamma))$ is called a local distribution of the function over γ with respect to $\mathbf{a}$, or a $(\gamma, \mathbf{a})$-local distribution.

Let γ and $\gamma^{\perp}$ be two orthogonal subcubes of F_q^n and $\mathbf{a}$ form its intersection. Then the $(\gamma, \mathbf{a})$-local distribution uniquiely determines the $(\gamma^{\perp}, \mathbf{a})$-local distribution. The main results are following. First, we obtain the formula for the generating function of $(\gamma^{\perp}, \mathbf{a})$-local distribution in terms of the components of $(\gamma, \mathbf{a})$-local distribution. Then we obtain the explicit formula for the components of $(\gamma^{\perp}, \mathbf{a})$-local distribution $v_i^{\mathbf{a}}(\gamma^{\perp})$, $i = 0, 1, \ldots, n-k$. This formula can be useful for reconstructing an arbitrary centered function in a ball from its values over the corresponding sphere.